Responses to the reviews for the manuscript entitled, "Quadrature by Asymptotic Parity eXpansions (QPAX) for light scattering by high aspect ratio plasmonic particle"

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Main requests from Reviewer

1. *comparison to any other close-evaluation scheme, as [panel-based schemes of Helsing-Holst 2014].*
2. *clear statement that the error is asymptotic (O(eps) only, or higher?) and not convergent, and to show accuracy vs N somehow.*
3. *I feel strongly that bringing "parity" into this problem is a distraction: really parity is just a way of writing that the contribution from the self and nearby parts of the boundary both must be handled with an accurate quadrature schemes. This would be just as true if the object were banana-shaped (no symmetry).*

However, I understand if making all three changes is too hard. The authors also should read recent slender-body theory literature for the Stokes BVP (Mori, Ohm, etc) and borrow those ideas instead of parity.

Response. We have addressed points (2) and (3) in the proceeding, and will work on implementing comparisons for the talk. We thank the reviewer for the comments and the reference.

Other fix

1. *For a reasonable eps such as 1e-2, what N is needed to get reasonable accuracy in PQR? Surely N*
2. *Sentence 1 of Sec 1: "some property" → "a constant material property"*
3. *"mirror points" would be better decsribed as the reflection about the y-axis.*
4. *conducts to → gives*
5. *Fig. 1 why does PQR for, say, eps>0.3, not give many correct digits? (N=128 should be sufficient for non-singular geometries).*
6. *statements like "ill-posedness occurs with parity" are confusing (and untrue in my opinion) since one could easily have a non-symmetric narrow obstacle, for which parity has no meaning. It is simply an issue of accurate close-evaluation in forming the Nystrom matrix.*
7. *What is a "classical" penetrable ellipse and why is it in italics? (why can positive permittivity ratio be done analytically by Mathieu but not the plasmonic case?)*

Response.

1. When ε becomes "subgrid", this is where the nearly singular behavior drastically affects PQR. So for $\varepsilon = 10^{-2}$, one requires $N \geq \frac{\pi}{\varepsilon} \approx 314$ quadrature points.
2. Done
3. Done
4. Done
5. Indeed it may look strange that the error even for PQR doesn't decrease for larger ε. This may be coming from the sensitivity while truncating the Mathieu series (see reference [2] for more details) for the analytic solution that we need to handle better. The problem may come from the interior solution, decomposed on Mathieu functions of the first kind (whose series convergence is more sensitive than the one with Mathieu functions of the third kind used for the exterior solution).
6. We have clarified what we mean by parity and provided more comments on the ill-posedness. Parity simply refers to even/odd properties along the reflections points, and this highlighted nearly-singular behavior also arises for non-symmetric particles such as banana-shaped ones (one can define local parity there, as defined in the proceeding). More details are provided in reference [2].
7. Solutions are computed as Mathieu function series using Python, and radial Mathieu functions in SciPy do not accept negative parameters.