Plasmonic cavity modes with sign changing permittivity

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Abstract

We study a 2D dielectric cavity with a metal inclusion. The permittivity ϵ of the metal depends on the frequency ω and, in a given frequency range, the metal can be (almost) dissipationless ($|Im(\epsilon(\omega))| \ll |Re(\epsilon(\omega))|$) and such that $Re(\epsilon(\omega)) < 0$. We look for the cavity resonance values ω. Due to the dependence of ϵ with respect to ω, this is a non linear eigenvalue problem. Below we consider mainly the linearized-problem where this dependence is frozen. Under some conditions on ϵ and the inclusion's geometry, the linearized-problem principal operator is self-adjoint with compact resolvent. Besides when the inclusion has corners adding to the fact that ϵ is sign changing at the boundaries between the metal and the dielectric, self-adjointness and compactness of the resolvent may be no longer true. This is due to very singular phenomena at the corners, which require a new functional framework for the theoretical analysis, and a specific numerical treatment. The non linear case which requires a fixed point algorithm is briefly discussed.

Introduction

Let's consider a cavity Ω, $\overline{\Omega} = \overline{\Omega_1} \cup \overline{\Omega_2}$, with a dielectric material Ω_1, and a metal inclusion Ω_2. Let's call the interface $\Sigma = \overline{\Omega_1} \cap \overline{\Omega_2}$. We study the following eigenvalue problem :

$$(1) \quad \begin{cases} \text{Find } u \neq 0,\, \omega \in \mathbb{R}\, s.t. : \\ -\text{div}(\frac{1}{\epsilon(\omega)}\nabla u) = \omega^2 \mu(\omega) u \quad \text{in } \Omega \\ u = 0 \quad \text{on } \partial\Omega \end{cases}$$

where the electric permittivity $\epsilon(\omega)$ is a non linear real valued function of the frequency ω.

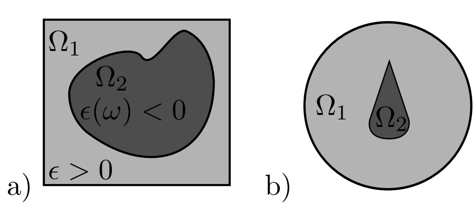


Figure 1: : Examples of a cavity. The configuration b) will provide support for numerical illustrations.

For simplicity we consider the linearized eigenvalue problem in ω, which consists in replacing $\epsilon(\omega)$ by ϵ in (1) and, we focus our attention on the case where $\epsilon < 0$ in the inclusion. More precisely, we take ϵ and μ piecewise constant functions, $\mu > 0$ almost everywhere and ϵ sign changing at the interface Σ. Let's define the principal operator :

$$A : \left| \begin{array}{c} D(A) \subset L^2(\Omega) \longrightarrow L^2(\Omega) \\ u \mapsto -\frac{1}{\mu}\text{div}(\frac{1}{\epsilon}\nabla u) \end{array} \right. \quad \text{with}$$

$D(A) = \{u \in H^1_0(\Omega), \frac{1}{\mu}\text{div}(\frac{1}{\epsilon}\nabla u) \in L^2(\Omega)\}$ and consider the weighted L^2 inner product $(u, v) \longmapsto \int_\Omega \mu uv\, d\Omega$. Thus our goal is to find the eigenvalues of A. For a given ϵ and depending on the interface Σ, the operator A can be self-adjoint or not. The next part is dedicated to solving the self-adjoint case, the one after that to solving the non self-adjoint case. We also present some computations in each section, with a specific numerical treatment in the second one because of particular phenomena near the corners.

1 The self-adjoint case

When $\epsilon > 0$ almost everywhere, the operator A is self-adjoint with compact resolvent (noted for simplicity SC. in the rest of the paper). The eigenvalues are positive with finite multiplicity and tend to infinity.

When ϵ changes sign, one can still have SC. properties for A under some conditions on ϵ and the interface Σ (precised below)[1], [2] : the eigenvalues then consist in two sequences of real numbers with finite multiplicity tending respectively to $\pm\infty$ (see fig.5a).

For a regular interface Σ (fig.1a), A is SC. if and only if $\frac{\epsilon|_{\Omega_2}}{\epsilon|_{\Omega_1}} \neq -1$. When Σ has corners (fig.1b), the operator is SC. if and only if $\frac{\epsilon|_{\Omega_2}}{\epsilon|_{\Omega_1}}$ doesn't belong to a critical interval containing -1, which is determined by the sharpest corner of the interface.

In this case, we have made computations with standard Finite Element for the geometry fig.1b. We observe stability of the results with respect to the mesh size (see fig.2). In fig.3 we observe that the modes are confined outside or inside the metal inclusion depending on the eigenvalues' sign.

Nodes	3469	7325	32049	132001
1st ev > 0	1.4836	1.4837	1.4807	1.4805
1st ev < 0	-4.083	-4.0771	-4.0762	-4.0758

Figure 2: : First positive and negative eigenvalues (of smallest modulus) for several mesh sizes.

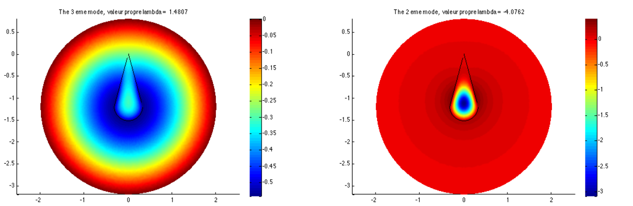


Figure 3: : First positive (left) and negative (right) modes of the SC. operator, associated to the two previous eigenvalues.

2 The non self-adjoint case

For $\frac{\epsilon|_{\Omega_2}}{\epsilon|_{\Omega_1}}$ chosen in the critical interval (excluding $\frac{\epsilon|_{\Omega_2}}{\epsilon|_{\Omega_1}} = -1$), due to singular phenomena at the corners, the SC. properties of A are no longer satisfied in the classical functional framework. In this case, the spectrum of A is the whole complex plane.

In [2], [3] (see also [4]) is given an extension of the operator A which has a compact resolvent, called A^+. It is defined by $D(A^+) = D(A) \oplus \text{span}\{s_1^+, \cdots, s_k^+\} \subset L^2(\Omega)$, where $s_1^+, \cdots, s_k^+,\ k \in \mathbb{N}$ are singular functions at k corners ($k \leqslant$ total corners' number of the interface Σ) that don't belong to H^1. These singularities, selected by a limiting absorption principle (see [3]), can be interpreted as waves propagating along the interface Σ towards the k corners : they are called black-hole plasmonic waves.

Numerically, there is no Finite Element convergence due to these black-hole waves. Thus, in order to capture confined plasmonics waves near the corners, a specific numerical treatment is performed. We operate an original use of PMLs (Perfectly Matched Layers) : by the Euler change of variables $(r, \theta) \longmapsto (log(r), \theta)$ we transform a disk centered at a corner into a waveguide [2] which we can troncate with PMLs. The PMLs domain corresponds to the small hole at the corner in fig.4. Numerical results confirm that the PMLs' method is efficient to ensure the stability of the Finite Element approximation. The A^+'s spectrum contains complex eigenvalues which clearly proves its non self-adjointness. We can prove that the eigenvalues belong to $\{z \in \mathbb{C}\, s.t.\, \text{Im}(z) \leq 0\}$, which is numerically almost satisfied (see fig.5b).

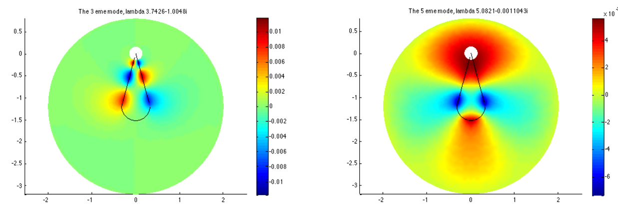
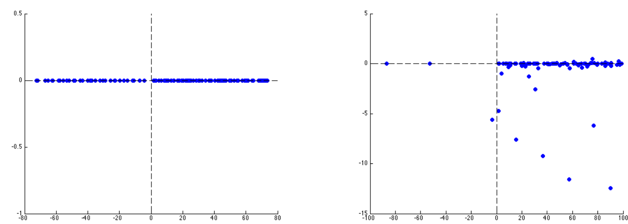


Figure 4: : Third and fifth modes of operator A^+ (associated to the smallest eigenvalues in modulus : $\lambda_3 = 3.7426 - 1.0046i$ and $\lambda_5 = 5.0821 - 1.1043 \times 10^{-3}i$).



(a) (b)

Figure 5: : (a) Spectrum of the SC. operator in the complex plane. (b) Spectrum of operator A^+ in the complex plane. (The scales are different.)

3 Conclusion/Ongoing work

Once we are able to understand the linearized eigenvalue problem, we could solve in principle our starting non linear problem (1). The cavity modes could be obtained for instance with a fixed point algorithm.

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